e.m. waves

Some useful (insightful) derivations

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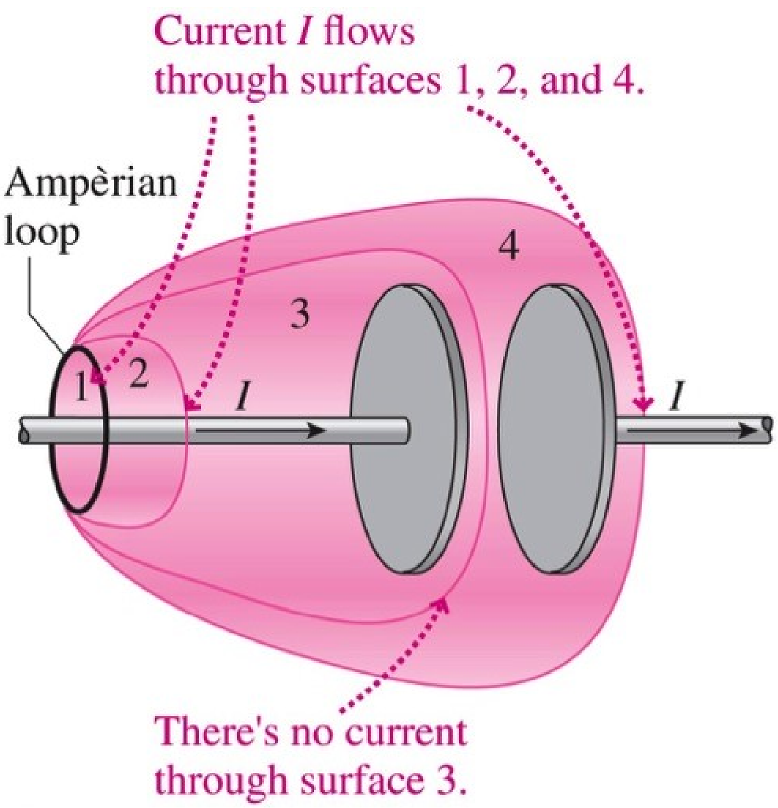
Displacement current (derivation)

Current resulting in change (displacement) of electric field



Total current in Ampere's loop law is sum of conduction current (i_C) and displacement current (i_D)

$$\oint B \bullet dl = \mu_o i_c + \mu_o \varepsilon_o \frac{d\phi_E}{dt}$$

Ampere-Maxwell law

With the introduction of displacement current the electromagnetic phenomena are more *symmetric* as changing magnetic field results in electric field and changing electric field results in magnetic field.

<u>Energy density due to ***E*** is equal to average energy density due to ***B***</u>

$e.m.$ waves consists of sinusoidal electric field oscillations therefore average energy density (over one complete cycle) due to ***E*** is

$$U_E = \frac{1}{2} \times \frac{\varepsilon_o E_o^{\ 2}}{2}$$

$$E_o = c_o B_o \quad \text{therefore}$$

$$U_E = \frac{1}{2} \times \frac{\varepsilon_o c_o^{\ 2} B_o^{\ 2}}{2}$$

$$c_o^{\ 2} = \frac{1}{\mu_o \varepsilon_o} \quad \text{therefore}$$

$$U_E = \frac{1}{2} \times \frac{B_o^{\ 2}}{2\mu_o}$$

this is same as average energy density (over one complete cycle) due to ***B***

Poynting vector

Rate at which energy passes through a unit surface area perpendicular to the direction of wave propagation is given by the *Poynting vector* (S).

$$\overline{S} = \frac{1}{\mu_o}\overline{E}\times\overline{B} = \overline{E}\times\overline{H}$$

Note : E and B in above relations are instantaneous values.

SI unit is W m^2

For a plane electromagnetic wave, $\boldsymbol{E}$ is perpendicular to $\boldsymbol{B}$ and $E = Bc$ therefore magnitude of Poynting vector is given by

$$S = \frac{EB}{\mu_o} = \frac{B^2 c}{\mu_o} = \frac{E^2}{c\mu_o}$$

Time average of S over one or more cycles is called the wave intensity (I)

$$I = S_{\text{<avg>}} = \frac{B_o^{\,2} c}{2\mu_o} = \frac{E_o^{\,2}}{2c\mu_o}$$

Note : E_o and B_o in above relations are peak values

Energy transfer is due to the radiation emitted by source. If we consider a spherical enclosure of radius r with source of light in it then

$$\frac{P}{4\pi r^2} = I = \frac{B_o^{\,2} c}{2\mu_o} = \frac{E_o^{\,2}}{2c\mu_o}$$

The above set of relations are used frequently in Boards and competitive examinations

Relation between intensity of *e.m.* wave and peak fields (alternate approach)

Energy density due to only the electric field (E) component of $e.m.$ wave is

$$\frac{U}{vol} = \frac{1}{2}\varepsilon_o E^2$$

Considering *rms* value of E due to sinusoidal nature of $e.m.$ wave, we get

$$\frac{U}{vol} = \frac{1}{2}\frac{\varepsilon_o {E_o}^2}{2}$$

As contributions of E and B densities are same, total energy density is given by

$$\frac{U_{\text{total}}}{vol} = \frac{\varepsilon_o {E_o}^2}{2} \quad \text{--- (i)}$$

Intensity is given by

$$I = \frac{P}{A}$$

$$I = \frac{U}{At}$$

$$I = \frac{U}{At} \times \frac{c}{c}$$

$$I = \frac{Uc}{Al}$$

Al is volume enclosed in a unit time interval due to propagation of wave

$$I = U_{\text{density}} \times c$$

Using relation (i)

$$I = \frac{1}{2} c \varepsilon_o {E_o}^2$$

Intensity variation depending on nature of sources

Intensity is given by the amount of energy transferred per unit area per unit time. Intensity decreases with increase in distance from the source.

$$I = \frac{P}{A}$$

- For a point source

$$I = \frac{P}{4\pi r^2}$$

- For a thin linear source

$$I \propto \frac{1}{r}$$

- For a planar source

$$I \propto r^0$$